MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 97 No. 5

MEETING OF 1937 MARCH 12

Dr. H. Spencer Jones, President, in the Chair.

The President announced that owing to unforeseen difficulties, which had resulted in the authorities concerned refusing to pass the plans, the projected scheme of stands along the front of Burlington House to view the Coronation procession had had to be abandoned, and the ballot that had been held for seats was therefore void.

The election of the following Fellow by the Council was duly confirmed :—

Colin Kerr Grant, 2 Brookside, Cambridge.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Jiri Alter, University Observatory, 14 Čechova, Praha VII, Czechoslovakia (proposed by H. Slouka) ;

Donald Robert Barber, B.Sc., A.Inst.P., Lyndhurst, Salcombe Road, Sidmouth, Devon (proposed by D. L. Edwards) ;

David Stanley Evans, King's College, Cambridge (proposed by W. M. Smart) ; and

Theodore E. Sterne, Ph.D., Harvard College Observatory, Cambridge, Mass. (proposed by H. Shapley).

One hundred and twenty-nine presents were announced as having been received since the last ordinary meeting, including :—

A. E. Douglass, Climatic Cycles and Tree Growth. Vol. III. A Study of Cycles (presented by the Carnegie Institution of Washington).

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The Physical Society, London, Reports on Progress in Physics. Vol. III.
Centenary Commemorative Volume of London University, 1836–1936. 1936 June 27–July 3.
W. H. White, A Complete Physics, written for London Medical Students and general use.

A film lent by Dr. H. D. Curtis was shown, exhibiting Recent Researches on Solar Prominences, made with the new Solar Tower of the McMath-Hulbert Observatory of the University of Michigan.

THE STELLAR PROBLEM OF THREE BODIES: IV. PERTURBATIONS IN THE SYSTEM ξ URSÆ MAJORIS.

Ernest W. Brown, Sc.D., F.R.S.

1. The system ξ Ursæ Majoris consists of two pairs of stars, the distances between the components of each pair being small compared with that of one pair from the other. The first pair is a very close one, the components revolving round one another in a period of 3·98 days. The period of revolution of the components of the second pair about one another is 1·832 years, while that of the revolution of the two pairs about each other is 59·863 years. It is the perturbing effect of the first pair, treated as a single body with mass equal to the sum of their masses, on the relative motion of the components of the second pair that is the subject of this paper. The perturbations in the motion of the components of the first pair about one another are quite insensible to observation. The system is almost ideal for the illustration of the developments set forth in Papers II and III. Both eccentricities are large, having present values of ·531, ·413, and the inclination of the planes of the orbits is also large, its value at epoch being $37°{\cdot}8$.

2. The calculation shows that the apse-node terms, the principle one of which has twice the angle between the apse and node as argument with a period of 1930 years, cause a variation of the apparent eccentricity from ·18 to ·54 and of the inclination from 37° to 47°; there are corresponding variations of the longitudes of the apse and node about their mean rotating vectors. The principal long-period terms in the elements have a period of thirty years, with coefficients of the order of 1°.

3. The system of elements used below was furnished to me by Professor H. N. Russell in a letter which I have his permission to quote. Indeed this series of papers had its inception in a conversation with him in which he pointed out that in the great majority of stellar cases of the problem of three bodies, the ratio of the periods is small. This fact suggested that the problem be treated from this point of view rather than from that which is usual in satellite and planetary theories. The chief difference is the possibility of obtaining considerable accuracy with large values of

the eccentricities and inclination without extensive calculation. Professor Russell writes :

"The star ξ Ursæ Majoris ($11^h\ 12^m.9$, $+32°\ 6'$) is a visual binary, with component A 4·41 G_0 and B 4·87 G_0, each of which is a spectroscopic binary. The elements of the visual orbit are accurately known *—$a'' = 2''.536$. $P = 59^y.863$. $e = 0.413$. $\omega = 127°\ 18'$. $i = +57°.20$. $\Omega = 101°.40$. $T = 1935.027$. The elements of the orbit of A about the centre of gravity of the spectroscopic pair Aa † are $a \sin i = 62.2 \times 10^6$ km. $P = 1^y.832$. $e = 0.531$. $\omega = 320°.0$. $T = 1909.752$.

"Photographic observations of the wide pair show an oscillation of A with this period, from which van den Bos ‡ derives the additional elements, for the orbit about the centre of gravity, $a'' = 0''.051$, $i = -84°.5$, $\Omega = 309°.4$.

"The pair Bb has so short a period ($3^d.980$) that its action upon Aa may be replaced by that of a single mass.

"The trigonometric parallax is $0''.138 \pm 0''.006$, according to Schlesinger's Catalogue.§ Berman's spectroscopic study || gives the parallax $0''.136 \pm 0''.012$. With Schlesinger's parallax, the mass of the system is 1·745 times the Sun's, and the mean distance of A from the centre of gravity of Aa is $0''.059$—probably a more accurate value than that derived from the photographs.

"Berman finds for the mass-ratio $M_{Bb}/M_{Aa} = 0.77$—with considerable uncertainty. The ratio $M_{Bb}/(M_{Aa} + M_{Bb}) = 1/2.29$ has been adopted. With this value, the mean distance a' of a from A comes out $0''.204$, whence $M_a/M_A = 0.41 = M/E$, and $a'/a = 1/12.4$. Also $(E - M)/(E + M) = 0.42$.

"In the pair Bb, $a \sin i = 2.76 \times 10^5$ km., whence $M_b^3 \sin^3 i/(M_B + M_b)^2 = 5.4 \times 10^{-5}$. Since $M_B + M_b = 0.76$, $m_b \sin i = 0.041$. Unless i is small for this pair (which seems improbable, as it is large for the other two) m_b must be the smallest known stellar mass."

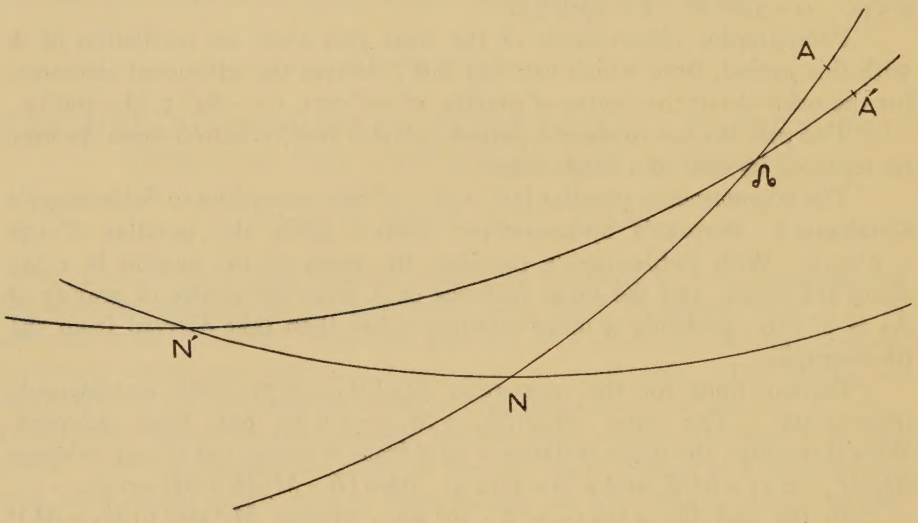
4. *Interpretation of Elements deduced from Observation.*—In satellite and stellar problems we have to deal with periods of three orders of magnitude fairly well separated from one another when the ratio of the periods is small. The *short periods* are those of the order of magnitude of the period of revolution of the close pair and shorter. The *long periods* are those of the order of magnitude of the period of revolution of the outer star and of a few submultiples. The *apse-node periods* are very much longer. If m be the ratio of the periods of revolution, the orders of magnitude of the three sets of periods are in the ratios $1 : m : m^2$.

Four different kinds of elements will be recognized. The true *osculating elements* which need no definition here. The *long-period elements* which are the osculating elements without their short-period variations. As the latter are negligible here the long-period elements are the same as the osculating elements. The *apse-node* elements are freed from short- and long-period variations and contain only the mean motions of the apse and

* Aitken, *The Binary Stars* (New York, 1935), p. 285.
† *Ibid.*, p. 294. ‡ *Ibid.*, p. 285.
§ Yale University Observatory, 1935.
|| *Lick Observatory Bulletin*, **15**, 109, 1930.

node and the apse-node terms. Finally, we have the *mean elements* which are constants.

In the present case, the three sets of periods have the orders 2 years, 60 years and 2000 years. As the elements given above are averages taken over an interval of the order of half a century they are probably nearly free from the long-period terms and can be considered as apse-node elements. A fresh discussion of the observations with the long-period terms and the small changes in the apse-node terms from the epoch values inserted would give



the true apse-node elements at the epoch, these being the elements needed for use at the present time and for the next century or so.

It is perhaps advisable to emphasize the point that throughout this series of papers reference is made to variations of the elements and not to variations in the co-ordinates.

5. The expression of the long-period perturbations in terms of f', the true anomaly of the outer orbit is unusual. But the abbreviation of the formulæ is so great when the eccentricity e' of the orbit is large that no apology is needed for the new form. Tables giving the value of f' for any values of the mean anomaly and e' are available, so that the calculation presents no additional difficulty.

I have endeavoured to obtain the numerical results correctly, applying such checks as were available. But as an expert computer could repeat them in a week or so from the formulæ given, an error would not be serious. And further, any changes due to changes in the given elements which he might wish to make can be easily carried out with the detailed formulæ given in Paper II and below.

6. In order to apply the formulæ previously obtained it is necessary to refer the given elements to the plane of motion of the outer star m_3.

The negative inclination of the orbit of m_2 about m_1 is changed to a positive one by adding 180° to the longitude of the node. Let accented letters refer to the elements of the orbit of m_3.

In the figure NN' is the trace of the given plane of reference; $N'A'$, NA those of the orbits of m_3, m_2. From the set of elements given in paragraph 3, we have $N'N = 28°{\cdot}0$, $\Omega N'N = 57°{\cdot}2$, $\Omega NN' = 95°{\cdot}5$. With the aid of the formulæ for a spherical triangle, sides a, b, c and inner angles A, B, C,

$$\cos C = -\cos A \cos B + \cos c \sin A \sin B, \quad \frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C},$$

we deduce

$$N'\Omega N = i = 37°{\cdot}8, \qquad N'\Omega = 49°{\cdot}75, \qquad N\Omega = 40°{\cdot}14.$$

Thence, if we use N' as the origin of reference,

$$\Omega = 49°{\cdot}75, \qquad \omega = \varpi - \Omega = 99°{\cdot}86, \qquad \varpi' = 127°{\cdot}18.$$

In the notation of Papers II, III, $\Omega = h$, $\omega = g$.

Further, from the same source, putting $L = 1$, we have

$$\frac{P}{P'} = \frac{n'}{n} = {\cdot}0306, \qquad \lambda = \frac{m_3}{m_1 + m_2 + m_3} = {\cdot}436,$$

$$k_1 = \frac{3}{8}\lambda\frac{n'}{n}(1 - e'^2)^{-\frac{3}{2}} = {\cdot}00662,$$

$$k_2 = \frac{3}{8}\lambda\frac{n'}{n}(1 + \tfrac{2}{3}e'^2)(1 - e'^2)^{-\frac{3}{2}} = {\cdot}00738,$$

and, at the epoch,

$$G = (1 - e^2)^{\frac{1}{2}} = {\cdot}8474, \qquad H = G\cos i = {\cdot}6696.$$

7. *The Long-period Terms in the Elements.*—The major-axis, and therefore n, have no sensible variations. Those of the other elements are calculated from the formulæ given in paragraph 6, Paper II (*M.N.*, **97**, 62–66). The elements used in the calculation are, as stated above, the *apse-node* elements for the present epoch. The coefficients should be recalculated every thirty years or so with the apse-node values suitable to the corresponding dates given below: interpolation will furnish the coefficients for intermediate dates.

The following table gives the results obtained for the coefficients of the sines and cosines of f', $2f'$, $3f'$ (the only multiples of f' present) in the expressions for the elements given in the heading:—

	δe	$\delta\varpi$	δi	$\delta\Omega$	$\delta(nt+\epsilon)$
	°	°	°	°	°
$\sin f'$	−0·16	+0·33	+0·32	−0·73	−0·38
$\cos f'$	− ·20	− ·73	+ ·29	− ·17	+ ·08
$\sin 2f'$	− ·48	+1·38	+ ·88	− ·29	− ·31
$\cos 2f'$	− ·48	−1·78	+ ·70	− ·41	+ ·19
$\sin 3f'$	− ·07	+ ·19	+ ·12	− ·04	− ·04
$\cos 3f'$	− ·06	− ·25	+ ·10	− ·06	+ ·03

8. *The Calculation of the Apse-node Terms.*—Since the formulæ for the apse-node terms are scattered through Paper III (*M.N.*, **97**, 116–127), and since the calculation is made by the method outlined very briefly in § 10, all of the formulæ needed will be given below, so that the method may be applied easily to any similar case. Some slight changes of notation occur.

9. The first step is the calculation of G in terms of g from the equation $R = \text{const.}$ Here

$$R = -A - \tfrac{3}{2}H^2 + B\cos 2g = C - \tfrac{3}{2}H^2,$$

where

$$A = -\tfrac{5}{2}\frac{H^2}{G^2} - \tfrac{1}{2}G^2 + \tfrac{1}{2}k_2 H\left(-35 + 33G^2 - 15\frac{H^2}{G^2} + 17H^2\right),$$

$$B = \tfrac{5}{2}(1 + 3Hk_2)\left(1 - \frac{H^2}{G^2}\right)(1 - G^2), \qquad C = \text{const.}$$

In order to exhibit to some extent the degree of approximation reached in confining the work to the two lowest powers of $m = n'/n$, I shall give most of the results for the cases $k_2 = 0$ and $k_2 = \cdot 00738$ found above.

Substituting the given values of the elements in the expression for R, I find

$$C = +1\cdot 671 \quad (k_2 = 0), \qquad C = +1\cdot 632 \quad (k_2 \neq 0).$$

In making use of the method of harmonic analysis to deduce a Fourier series, we give to the variable any convenient values ; later the corresponding values of the function are analysed to obtain the series. Five values appear to be sufficient here to obtain the needed accuracy. They are chosen to be $2g = 0°$, $180°$, $90°$, $60°$, $120°$. These are convenient because all the values of $\cos 2ig$ are then 0, ± 1, or $\pm \cdot 5$.

For each value we evidently have a quadratic in G^2 to solve. As $0 < G^2 < 1$, there will be found to be no ambiguity. The special values found for G are :

$2g =$	0	180	90	60	120
$k_2 = 0$	$+\cdot 9875$	$+\cdot 8408$	$+\cdot 9640$	$+\cdot 9811$	$+\cdot 9091$
$k_2 \neq 0$	$+\cdot 9822$	$+\cdot 8415$	$+\cdot 9539$	$+\cdot 9739$	$+\cdot 9029$

With these the special values of $-1 \div \partial R/\partial G$ are :

$k_2 = 0$	$+\cdot 2532$	$+\cdot 4003$	$+\cdot 6500$	$+\cdot 3761$	$+\cdot 7728$
$k_2 \neq 0$	$+\cdot 2403$	$+\cdot 3754$	$+\cdot 5485$	$+\cdot 3462$	$+\cdot 6291$

The analysis of these and of any of the cosine series developed below is made by the following formulæ, the example being that of the last function for $k_2 = 0$.

If the five special values be denoted by f_1, f_2, f_3, f_4, f_5 and the series by

$$a_0 + a_1\cos 2g + a_2\cos 4g + a_3\cos 6g + a_4\cos 8g,$$

we have the following results :—

$$\begin{aligned}
2a_0 + 2a_2 + 2a_4 = f_1 + f_2 &= +\ \cdot 6535, & 4a_2 &= -\cdot 6465,\\
2a_0 - 2a_2 + 2a_4 = 2f_3 &= +1\cdot 3000, & a_2 &= -\cdot 1616,\\
2a_0 - a_2 - a_4 = f_4 + f_5 &= +1\cdot 1489, & a_2 + a_4 &= -\cdot 1651,\\
3a_2 + 3a_4 &= -\ \cdot 4954, & a_4 &= -\cdot 0035,\\
2a_0 = \cdot 6535 + \cdot 3302 &= +\ \cdot 9837, & a_0 &= +\cdot 4919,\\
2a_1 + 2a_3 = f_1 - f_2 &= -\ \cdot 1471, & 3a_1 &= -\cdot 5438,\\
a_1 - 2a_3 = f_4 - f_5 &= -\ \cdot 3967, & a_1 &= -\cdot 1813,\\
& & a_3 &= +\cdot 1077.
\end{aligned}$$

The series for $-1 \div \partial R/\partial G$, $k_2 \neq 0$, is

$$+\cdot4278 - \cdot1393 \cos 2g - \cdot1203 \cos 4g + \cdot0718 \cos 6g + \cdot0004 \cos 8g.$$

The equation for g is

$$-dg \div \partial R/\partial G = d\tau.$$

Substituting and integrating we obtain series which may be written

$$g = +2\cdot033\tau + \cdot1843 \sin 2g + \cdot0821 \sin 4g - \cdot0365 \sin 6g + \cdot0009 \sin 8g \qquad (k_2 = 0),$$

$$g = +2\cdot338\tau + \cdot1628 \sin 2g + \cdot0703 \sin 4g - \cdot0260 \sin 6g - \cdot0001 \sin 8g \qquad (k_2 \neq 0).$$

The arbitrary constant may be supposed to be included in τ. Denote the first terms by $g_m\tau$. The values of $g_m\tau$ at epoch derived from these formulæ are 98°·6, 99°·2.

10. The series has to be reversed to obtain g in terms of $g_m\tau$. This may be done by the use of Lagrange's theorem which says that if

$$y = x + f(y),$$

then, with the notation f for $f(x)$,

$$y = x + f + \frac{1}{2!}\frac{d}{dx}f^2 + \frac{1}{3!}\frac{d^2}{dx^2}f^3 + \ldots$$

To a sufficient degree of approximation this can be written

$$y = x + f + \tfrac{1}{2}\frac{d}{dx}\left\{\frac{f^2}{1 - f'} + f^3 f''(\tfrac{1}{3} + f')\right\},$$

where f', f'' are the first and second derivatives of $f(x)$ with respect to x. This is a convenient form for the use of harmonic analysis since the portion under the derivative is a cosine series which is zero for the special values 0, 180 of the argument.

In this way I obtain

$$g = g_m\tau + \cdot1669 \sin 2g_m\tau + \cdot1187 \sin 4g_m\tau + \cdot0158 \sin 6g_m\tau + \cdot0109 \sin 8g_m\tau, \qquad g_m = 2\cdot033, \quad k_2 = 0;$$

$$g = g_m\tau + \cdot1452 \sin 2g_m\tau + \cdot0975 \sin 4g_m\tau + \cdot0268 \sin 6g_m\tau + \cdot0303 \sin 8g_m\tau, \qquad g_m = 2\cdot338, \quad k_2 \neq 0.$$

11. The functions, previously expressed in terms of g, must now be expressed in terms of $g_m\tau$. To do this by harmonic analysis we give the same five special values to $2g_m\tau$. Expressing the coefficients in g in terms of angle by the use of the factor 57·295, I find, for $k_2 = 0$,

$2g_m\tau$	0°	180°	90°	60°	120°
$2g$	0°	180°	107°·32	87°·26	125°·85
G	·9874	·8408	·9319	·9671	·8971
e	·158	·541	·363	·254	·442
i	47°·3	37°·2	44°·1	46°·2	41°·7

and for $k_2 \neq 0$,

$2g$	0°	180°	103°·57	81°·07	127°·74
G	·9822	·8416	·9334	·9632	·8898
e	·188	·540	·359	·269	·456
i	47°·0	37°·3	44°·2	45°·9	41°·2

12. The equation for h is

$$\frac{dh}{d\tau} = \frac{\partial A}{\partial H} + 3H - \frac{\partial B}{\partial H} \cos 2g.$$

With this material given above we find for the special values of $dh/d\tau$,

$k_2 = 0$	−1·442	−2·531	−1·820	−1·573	−2·074
$k_2 \neq 0$	−1·491	−2·714	−1·835	−1·620	−2·154

These are analysed and integrated; the results are given below.

13. The formula for the periodic terms in the mean longitude is

$$\frac{d}{d\tau}\delta M = (G-1)\frac{dg}{d\tau} + (H-1)\frac{dh}{d\tau},$$

where the periodic terms on the right are alone retained. The first product is found by harmonic analysis and then integrated; the second product is integrated immediately since H is constant.

14. *Results.*—The series for e, i and the periodic parts of $\varpi = g + h$, of $\Omega = h$ and of $nt + \epsilon = M$, the mean longitude, are given in the following tables:—

Coeff. of $\cos pg_m\tau$, $\sin pg_m\tau$: $k_2 = 0$

p	0	1	2	3	4
e (cos)	+·349	−·190	−·006	−·001	+·008
ϖ (sin)		+17°·0	+6°·2	+1°·0	+0°·5
i (cos)	+43°·1	+ 4°·9	−0°·9	+0°·2	+0°·1
Ω (sin)		+ 7°·5	−0°·6	+0°·1	−0°·1
M (sin)		− 0°·4	−0°·1		−0°·6

Coeff. of $\cos pg_m\tau$, $\sin pg_m\tau$: $k_2 \neq 0$

	0	1	2	3	4
e (cos)	+·363	−·180	+·003	+·004	−·002
ϖ (sin)		+15°·5	+4°·8	+1°·6	+1°·7
i (cos)	+43°·1	+ 4°·5	−1°·0		
Ω (sin)		+ 7°·2	−0°·8	+0°·1	
M (sin)		− 0°·4	−0°·1	0°·0	−0°·6

The motions of the arguments are obtained by putting $\tau = k_1 f' = \cdot 00662 f'$. The non-periodic parts are therefore

	ϖ	Ω
$k_2 = 0$,	$\cdot 00103(f' - f_0') + \varpi_0$,	$-\cdot 01243(f' - f_0') + \Omega_0$,
$k_2 \neq 0$,	$\cdot 00251(f' - f_0') + \varpi_0$,	$-\cdot 01297(f' - f_0') + \Omega_0$,

and therefore for $2g_m\tau$,

$$\cdot 0269(f_0 - f_0') + 2\varpi_0 - 2\Omega_0, \qquad +\cdot 0310(f' - f_0') + 2\varpi_0 - 2\Omega_0$$

respectively. For $k_2 \neq 0$, these correspond to periods of 23900, 4620, 1930 years respectively. Since

$$f' = l' + \cdot 81 \sin l' + \cdot 20 \sin 2l' + \cdot 07 \sin 3l' + \cdot 03 \sin 4l',$$

and $2g_m k_1 = \cdot 0310 = 1^\circ \cdot 78$, where l' is the mean anomaly in the outer orbit, we have

$$2g_m k_1 f' = \cdot 0310 l' + 1^\circ \cdot 4 \sin l' + 0^\circ \cdot 4 \sin 2l' + 0^\circ \cdot 1 \sin 3l'.$$

The long-period terms thus introduced have the same orders of magnitude as those computed in § 7. It is, however, simpler to compute $2g_m k_1 f'$ for the dates required and to interpolate in the following table in which the dates correspond to equidistant values of f' and are therefore approximate.

	$2g_m\tau$	e	ϖ	i	Ω	δM
	°		°	°	°	°
1875	187·2	·536	149·9	37·4	56·2	−0·3
1905	192·8	·534	149·8	37·6	53·0	−0·4
1935	198·4	·531	149·6	37·8	49·8	−0·5
1965	204·0	·525	149·2	38·1	46·9	−0·5
1995	209·6	·518	148·5	38·4	43·6	−0·4

Any desired change in the mass function λ to a new value λ_1 is made by multiplying the coefficients of the long-period terms by λ_1/λ. If C_0 be the coefficient of an apse-node term when $k_2 = 0$ and C_1, its value when $k_2 \neq 0$, the new coefficient will be

$$C_0 + \frac{\lambda_1}{\lambda}(C_1 - C_0),$$

while the motion m_1 of an argument is changed to

$$\frac{\lambda_1}{\lambda} m_0 + \frac{\lambda_1^2}{\lambda^2}(m_1 - m_0).$$

These changes in the apse-node portions are valid only if λ_1 does not differ greatly from λ.

For comparison with observation it is unfortunate that at the present time the apse-node variations of e, ϖ, i are near their minima. On the other hand, the variation $\delta\Omega$ is about 3° in thirty years.

Yale Observatory:
1936 *December.*

Errata in Paper I

P. 58, line 7 from bottom. *For* w_i' *read* w_1'.

P. 59, line 3 from bottom. *For* $\frac{\mu}{a'}$ *read* $\frac{\mu}{a'^3}$.

P. 60, lines 13, 23. *For* w_i, w_i' *read* w_1, w_1'.

THE PHOTOELECTRIC LIGHT-CURVE OF ALGOL.

W. M. Smart, M.A., D.Sc.

(Communicated by the Director of Cambridge Observatory)

1. Since the autumn of 1932 Algol ($3^h\ 3^m.7$; $+40°\ 42'$; B8) has been kept under fairly regular observation with the photoelectric photometer * mounted on the Sheepshanks telescope. Throughout, δ Persei ($3^h\ 38^m.0$; $+47°\ 34'$; B5) has been used as the comparison star. The observations have been made by Mr. H. E. Green and the writer. Altogether the stars were observed on 84 nights ; some part of the principal minimum was observed on 23 nights, and of the secondary minimum on 12 nights. The telescope was out of action for part of the observing seasons of 1934 and 1935 owing to matters connected with the acquisition of a new photographic objective and the aluminising of the mirror at the coudé joint.

If m_β and m_δ denote the apparent magnitudes of Algol and δ Persei at the time of a particular observation, and M_δ, M_β the magnitudes corrected for atmospheric extinction, we have

$$\Delta M \equiv M_\delta - M_\beta = m_\delta - m_\beta + f(\sec z_\beta - \sec z_\delta),$$

where f denotes the extinction factor and z the zenith distance. The observations provide the values of $(m_\delta - m_\beta)$, which are corrected by means of the preceding formula, the corresponding values of f being deduced by methods described in earlier papers.† On the average it is possible to observe the bottom part of the principal minimum curve under satisfactory conditions only twice each month during the appropriate observing season. This may seem surprising. It is to be remembered, however, that the accuracy of which the instrument is capable is of the order of one or two thousandths of a magnitude (of course, with good observing conditions), and that in consequence the extinction corrections must be determined with corresponding accuracy. We denote the extinction correction by ΔC, so that

$$\Delta C = f(\sec z_\beta - \sec z_\delta).$$

Fig. 1 shows the values of ΔC (in units of $0^m.001$) plotted against the sidereal time, for values of the extinction factor, f, between $0^m.3$ and $0^m.6$. On a very good night the value of f is about $0^m.4$ for B-type stars ; on one exceptionally clear night it was measured as small as $0^m.36$. For most of the observations with which this paper deals, the seeing corresponded to an extinction factor ranging from $0^m.45$ to $0^m.55$. It will be inferred from the diagram that when Algol is east of the meridian the extinction corrections are comparatively small and that little accuracy is lost theoretically even

* Some details concerning the instrument and the procedure in making observations are to be found in *M.N.*, **95**, 644, 1935.

† W. M. Smart, *M.N.*, **94**, 115, 839, 1933.

if the estimated value of f has an uncertainty of $0^m.05$. But in practice the observations made when Algol is east of the meridian are restricted to sidereal times between 0^h and 3^h for the following reason. The difference

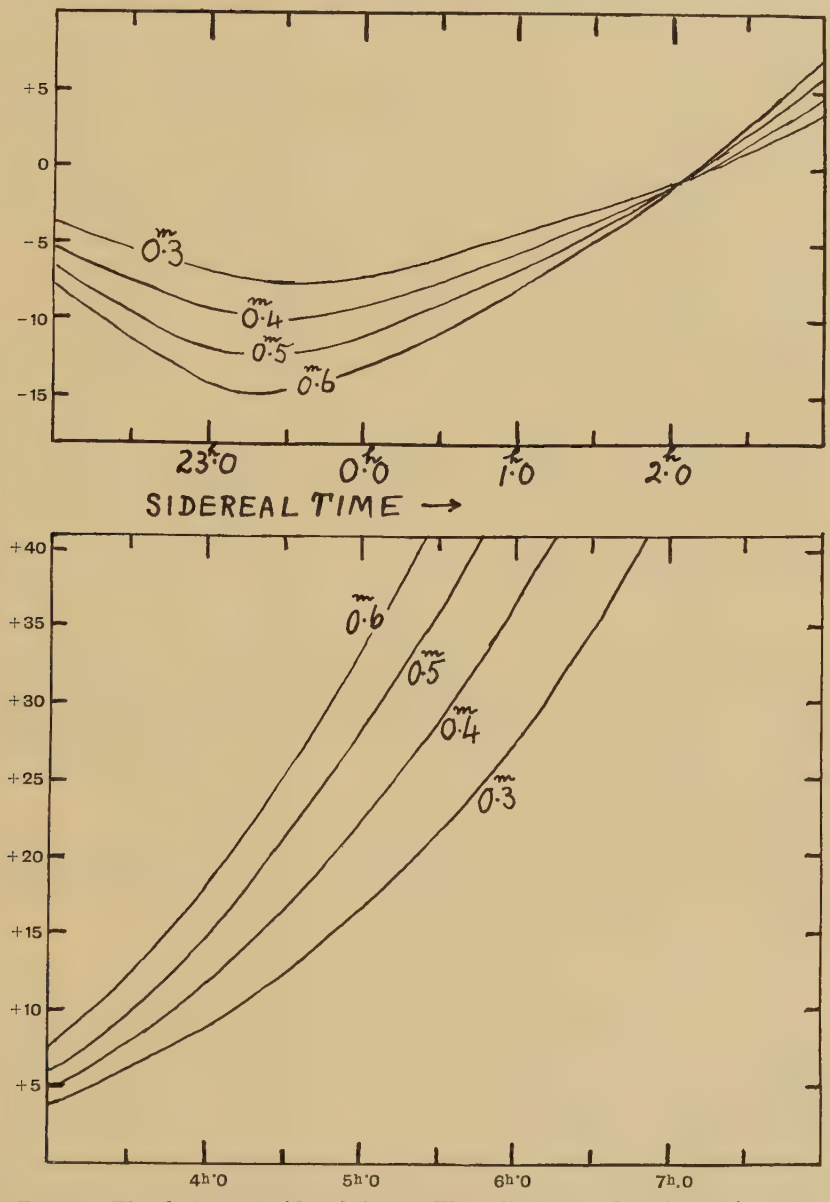


FIG. 1.—*The abscissæ are sidereal times. The ordinates are the values of* ΔC, *the extinction correction, in units of* $0^m.001$ *for the values* 0·3, 0·4, 0·5, 0·6 *of* f.

of magnitude between δ Persei and Algol is derived from a trio of observations made in the order: δ at sidereal time T_1; Algol at T; δ at T_2, the difference $T_2 - T_1$ being generally 6 to 10 minutes; from the stop-watch times observed for δ at T_1 and T_2 the stop-watch time at T is found by

simple interpolation. This latter value is then compared with the stop-watch time for Algol at T, and so the magnitude difference is deduced. If the zenith distance of δ is not altering rapidly, the interpolated value is not likely to be in error, given reasonable steadiness of the atmospherical conditions. If, however, the observations are made at sidereal times earlier than 0^h, the change in zenith distance of the comparison star is too rapid for interpolation to be entirely trustworthy. If Algol is west of the meridian, the observations are severely restricted owing to the rapidity with which the values of ΔC increase, and unless the extinction factor is determined with considerable accuracy, the value of the observations beyond sidereal time $4\frac{1}{2}^h$ (or perhaps 5^h on a really good night) rapidly depreciates.

The times referred to subsequently are all heliocentric, and for convenience 2,427,000 is subtracted from each of the Julian Day numbers.

2. Table I gives the details of observations made at different parts of the principal minimum curve; n denotes the number of periods since J.D. 2426999·0. Except for about half of the observations made in 1932, when the entries refer to the mean of pairs of single observations, the entries refer to single observations; a single observation is meant to imply the result obtained usually from 3 or 4 consecutive stop-watch readings for Algol combined with the interpolated stop-watch time for the comparison star. The phases have been calculated according to the procedure outlined later.

3. The period of Algol undergoes small fluctuations arising from the presence of a distant third component whose existence was first deduced from radial velocity measures of Algol independently by Belopolsky and Curtiss in 1908. The values of the orbital period of the distant companion, about the centre of mass of the system, as determined by Belopolsky,* Curtiss,† Schlesinger ‡ and McLaughlin,§ are 1·733, 1·899, 1·874 and 1·873 years respectively.

If A denotes Algol and G the centre of mass of the triple system, the value of $AG \sin i$ given by McLaughlin (*loc. cit.*) is $88 \cdot 10^6$ km., or, in light-time, 5^m approximately. The other relevant elements are:

ω	$= 93^\circ$.
T (time of periastron passage)	$= 1903 \cdot 38$.
e (eccentricity of A's orbit about G) =	0·26.

The perturbations of the binary orbit by the distant companion have been studied by R. A. Lyttleton.‖

4. From these considerations it follows that the period deduced from the light-curve is not quite constant. The average period, covering several weeks of observations of parts of the principal minimum curve, was derived as follows. The mean period was assumed, first of all, to be $2^d \cdot 86734$, and from the adopted epoch of J.D. 2426999·0 the phase for a given observation was calculated. From the observations the phase and the J.D. corresponding

* *Pulkowa Mitteil.*, 4, 171, 1911.
† *Ap. J.*, 28, 150, 1908.
‡ *A.A.S. Publ.*, 2, 128, 1912.
§ *Michigan Publ.*, VI, No. 2, 10, 1934.
‖ *M.N.*, 95, 42, 1934.

TABLE I

Observations on Principal Minimum Curve

n	Julian Date (heliocentric)	Phase	ΔM	J.D.	Phase	ΔM
	d	d	m			
4	12·4187	1·9484	+0·353	12·4996	2·0293	−0·256
	·4384	1·9681	+0·229	·5083	2·0380	−0·301
	·4925	2·0222	−0·210	·5229	2·0526	−0·363
12	35·4061	1·9952	−0·040	35·4393	2·0284	−0·273
	·4247	2·0139	−0·195			
13	39·3905	2·1121	+0·042	38·4720	2·1936	+0·648
	·4224	·1440	·321	·4764	·1980	·670
	·4284	·1498	·376	·4918	·2134	·727
	·4422	·1638	·470	·5114	·2330	·802
	·4474	·1690	·503	·5250	·2466	·822
	·4604	·1820	·582			
21	61·2386	2·0196	−0·245	61·4033	2·1843	+0·586
	·2486	·0296	·284	·4180	·1990	·642
	·2591	·0401	·315	·4315	·2125	·702
	·2896	·0706	−0·286	·4476	·2286	·757
	·3750	·1560	+0·391	·4639	·2449	·800
	·3882	·1692	·510	·4793	·2603	·814
22	64·2459	2·1593	+0·458			
27	78·3295	1·9050	+0·643	78·3984	1·9739	+0·142
	·3430	·9185	·567	·4209	1·9964	−0·071
	·3563	·9318	·462	·4301	2·0056	·123
	·3685	·9440	·389	·4379	·0134	·203
	·3825	·9580	·279	·4466	·0221	−0·239
	·3900	·9655	·189			
34	98·2598	1·7623	+0·832	98·3628	1·8653	0·786
	·2760	·7785	·836	·3728	·8753	·757
	·3111	·8136	·831	·3890	·8915	·702
	·3237	·8262	·825	·3955	·8980	·669
	·3398	·8423	·812	·4035	·9060	·639
35	101·2532	1·8881	+0·709	101·2762	1·9111	0·616
	·2616	·8965	·681	·2862	·9211	·554
	·2690	·9039	·647	·3025	·9374	·440
108	310·5068	1·8384	0·813	310·5258	1·8574	0·800
109	313·4994	1·9581	+0·293	313·5810	2·0397	−0·347
	·5082	·9669	·225	·5879	·0466	·373
	·5162	·9749	·143	·5944	·0531	·368
	·5249	·9836	+0·052	·5987	·0574	·350
	·5327	·9914	−0·015	·6055	·0642	·330
	·5486	2·0073	·142	·6123	·0710	·278
	·5652	·0239	·279	·6182	·0769	−0·224
	·5734	·0321	−0·320			

n	Julian Date (heliocentric)	Phase	ΔM	J.D.	Phase	ΔM
	d	d	m			
117	336·4183	1·9385	+0·419			
126	362·3352	2·0496	−0·336	362·4768	2·1912	+0·650
	·3407	·0551	·332	·4870	·2014	·690
	·3465	·0609	·323	·5225	·2369	·819
	·3591	·0735	−0·254	·5372	·2516	·820
	·4627	·1771	+0·575	·5510	·2654	·821
140	402·5116	2·0837	−0·169	402·5332	2·1053	+0·020
	·5168	·0889	·127	·5386	·1107	·071
	·5219	·0940	·087	·5438	·1159	·107
	·5276	·0997	−0·030	·5564	·1285	·228
150	431·2596	2·1586	+0·441	431·2986	2·1976	+0·676
	·2684	·1674	·513	·3174	·2164	·747
	·2812	·1802	·593	·3281	·2271	·772
	·2905	·1895	·647	·3678	·2668	·818
164	471·3534	2·1100	+0·043	471·3617	2·1183	+0·147
	·3574	·1140	·098	·3655	·1221	·178
253	726·4313	1·9974	−0·076			
261	749·3503	1·9779	+0·173	749·3761	2·0037	−0·085
	·3548	·9824	·110	·3809	·0085	·120
	·3718	·9994	−0·059	·3852	·0128	·140
382	1096·3948	2·0748	−0·271	1096·4733	2·1533	+0·403
	·4002	·0802	·215	·4773	·1573	·442
	·4058	·0858	·180	·4827	·1627	·475
	·4108	·0908	·136	·4869	·1669	·506
	·4230	·1030	−0·010	·4913	·1713	·523
	·4286	·1086	+0·008	·5122	·1922	·619
	·4333	·1133	·078	·5167	·1967	·640
	·4382	·1182	·122	·5208	·2008	·670
	·4431	·1231	·161	·5251	·2051	·690
	·4483	·1283	·211	·5335	·2135	·721
	·4647	·1447	·343	·5382	·2182	·739
	·4690	·1490	·375	·5424	·2224	·752
413	1185·3212	2·1145	+0·084	1185·3339	2·1272	+0·224
	·3276	·1209	·126			
414	1188·2788	2·2048	0·726			
508	1457·4647	1·8636	0·793	1457·4865	1·8854	0·719
	·4696	·8685	·792	·5324	·9313	·507
	·4812	·8801	·735	·5377	·9366	·453
517	1483·4172	2·0103	−0·176	1483·4811	2·0742	−0·277
	·4211	·0142	·226	·4878	·0809	·252
	·4248	·0179	·251	·4910	·0841	·214
	·4289	·0220	·282	·4945	·0876	·184

n	Julian Date (heliocentric)	Phase	ΔM	J.D.	Phase	ΔM
	d	d	m			
517	1483·4327	2·0258	−0·305	1483·4986	2·0917	−0·165
	·4365	·0296	·317	·5025	·0956	·125
	·4403	·0334	·344	·5061	·0992	·082
	·4442	·0373	·349	·5095	·1026	·059
	·4486	·0417	·374	·5128	·1059	·031
	·4777	·0708	·314	·5164	·1095	−0·009
526	1509·3341	2·1214	+0·110	1509·3445	2·1318	+0·206
	·3396	·1269	·174			

to the value, say, $\Delta M = 0^m{\cdot}65$ on the upward branch of the principal minimum, were found by interpolation. A similar calculation was made from observations made several periods later. As an illustration the results for the upward branch (1933 autumn) are shown in Table II.

TABLE II

Principal Minimum (Upward Branch)

ΔM	J.D.	n_1 (No. of Periods)	Phase	J.D.	n_2 (No. of Periods)	Phase
m						
+0·05	402·5365	140	2·1089	471·3536	164	2·1089
0·10	·5424	140	·1148	·3577	164	·1139
0·15	·5482	140	·1206	·3622	164	·1184
0·60	362·4666	126	·1818	431·2820	150	·1810
0·65	·4768	126	·1920	·2924	150	·1914

If the period, P, adopted is correct, the phase for any given value of ΔM should be the same in the 4th and the 7th columns. Let ϕ denote the correct phase corresponding to a given value of ΔM; let $P+\Delta P$ denote the true period which we regard as constant over the interval covered by a series of observations such as are shown in the table above. Then the J.D. is given by $\phi + n(P+\Delta P)$. If for convenience we write $\phi = \phi_0 + \Delta\phi$, where ϕ_0 is an approximate value of the phase, the equation of condition is

$$\Delta\phi + n\Delta P = \text{J.D.} - \phi_0 - nP.$$

The right-hand side is calculated from the known data and the assumed value of P. The equations of condition formed from the data of Table II are then as follows, the units on the right-hand of each equation being $0^d{\cdot}0001$:—

ϕ_0	$\Delta\phi + 140\Delta P =$		$\Delta\phi + 164\Delta P =$	
2·1000		89		98
·1100	140	48	164	39
·1100	140	106	164	84
·1800	126	18	150	10
·1900	126	20	150	14

Solving each pair of equations we obtain the results (in order) :—

TABLE III

ΔM	$\Delta\phi$	ΔP	Weight
m			
+0·05	36	+0·38	5
0·10	101	−0·38	5
0·15	235	−0·92	5
0·60	60	−0·33	2
0·65	51	−0·25	2

The weights were assigned according to the number of observations and their quality. The values of ΔP give a weighted mean of $-0{\cdot}30 \pm 0{\cdot}04$, so that the period of the light-curve for this particular series of observations is

$$2^{d}{\cdot}86731 \pm 0{\cdot}000004.$$

If the exact time of a principal minimum (corresponding to a minimum value of ΔM) is obtained, the phase for any given observation within the interval concerned can be readily found.

Assuming that the light-curve is symmetrical, we can also determine the phase, ϕ', at principal minimum when we have calculated the phases on the downward and on the upward branches of the curve for one or more definite values of ΔM. In this way the observations made in the autumn of 1932 give

$$\phi' = 2^{d}{\cdot}0491.$$

This value was adopted throughout as the basis of the light-curve constructed from all the observations.

5. The mean period of the light-curve can best be derived from observations separated by long intervals. The times of principal minimum were deduced from observations made on 1933 August 28 ($n = 109$) and 1936 November 10 ($n = 517$) as follows :—

$$\text{J.D. } 313{\cdot}5904\,; \quad 1483{\cdot}4560$$

respectively. The resulting period is

$$2^{d}{\cdot}867318,$$

which is as near the mean period as the present series of observations can be expected to give.

6. We have seen in section 4 that the period calculated from observations made in the autumn of 1933, at principal minima corresponding to values of n between 126 and 164, is $2^{d}{\cdot}86731$. A similar calculation, for a much larger number of equations of condition than were used in section 4, from observations made in the autumn of 1932, for values of n between 4 and 35, gave the period (regarded as constant during this interval) as

$$2^{d}{\cdot}86757 \pm 0{\cdot}000012.$$

The difference between this period and the mean period appears to be several times larger than would be expected from causes arising from the

TABLE IV

Normal Places

Phase	ΔM	No. of Observations	Phase	ΔM	No. of Observations
0·036	+0·848	11	1·9756	+0·153	3
·073	·852	8	1·9860	·081	2
·245	·845	2	1·9914	−0·015	1
·291	·839	7	1·9952	− ·040	2
·368	·846	2	1·9977	− ·069	3
·402	·845	7	2·0060	− ·117	4
·430	·841	8	2·0103	− ·176	1
·473	·836	15	2·0148	− ·219	5
·488	·831	4	2·0188	− ·248	2
·511	·827	9	2·0226	− ·259	4
·523	·828	4	2·0287	− ·286	6
·548	·825	10	2·0352	− ·328	4
·571	·822	10	2·0405	− ·345	3
·593	·826	15	2·0505	− ·360	4
·609	·829	7	2·0562	− ·341	2
·631	·825	6	2·0626	− ·326	2
·651	·827	3	2·0708	− ·293	3
·676	·837	2	2·0742	− ·267	3
·694	·836	5	2·0793	− ·230	3
·717	·829	7	2·0845	− ·187	3
·775	·836	3	2·0897	− ·153	4
·838	·850	2	2·0948	− ·106	2
·910	·862	4	2·0994	− ·056	2
0·962	·847	8	2·1042	−0·020	4
1·017	·848	9	2·1102	+0·031	5
1·055	·853	4	2·1144	·092	4
1·099	·845	2	2·1195	·129	4
1·183	·838	6	2·1235	·171	3
1·256	·840	4	2·1285	·219	4
1·327	·842	4	2·1442	·328	3
1·352	·832	4	2·1495	·376	3
1·362	·830	5	2·1546	·397	2
1·437	·834	5	2·1584	·447	3
1·480	·823	9	2·1634	·472	3
1·603	·828	6	2·1686	·510	7
1·641	·827	2	2·1820	·584	6
1·663	·817	1	2·1920	·644	5
1·707	·823	5	2·1980	·657	6
1·759	·829	5	2·2030	·694	4
1·776	·813	6	2·2131	·716	5
1·813	·822	6	2·2173	·743	2
1·8323	·819	4	2·2224	·752	1
1·8423	·812	2	2·2281	·762	3
1·8574	·800	2	2·235	·810	4
1·8658	·790	3	2·247	·822	2
1·8761	·760	3	2·252	·820	2
1·8801	·735	1	2·266	·820	3
1·8880	·712	5	2·312	·824	7
1·8972	·675	2	2·347	·827	1
1·9050	·643	4	2·386	·838	2
					27

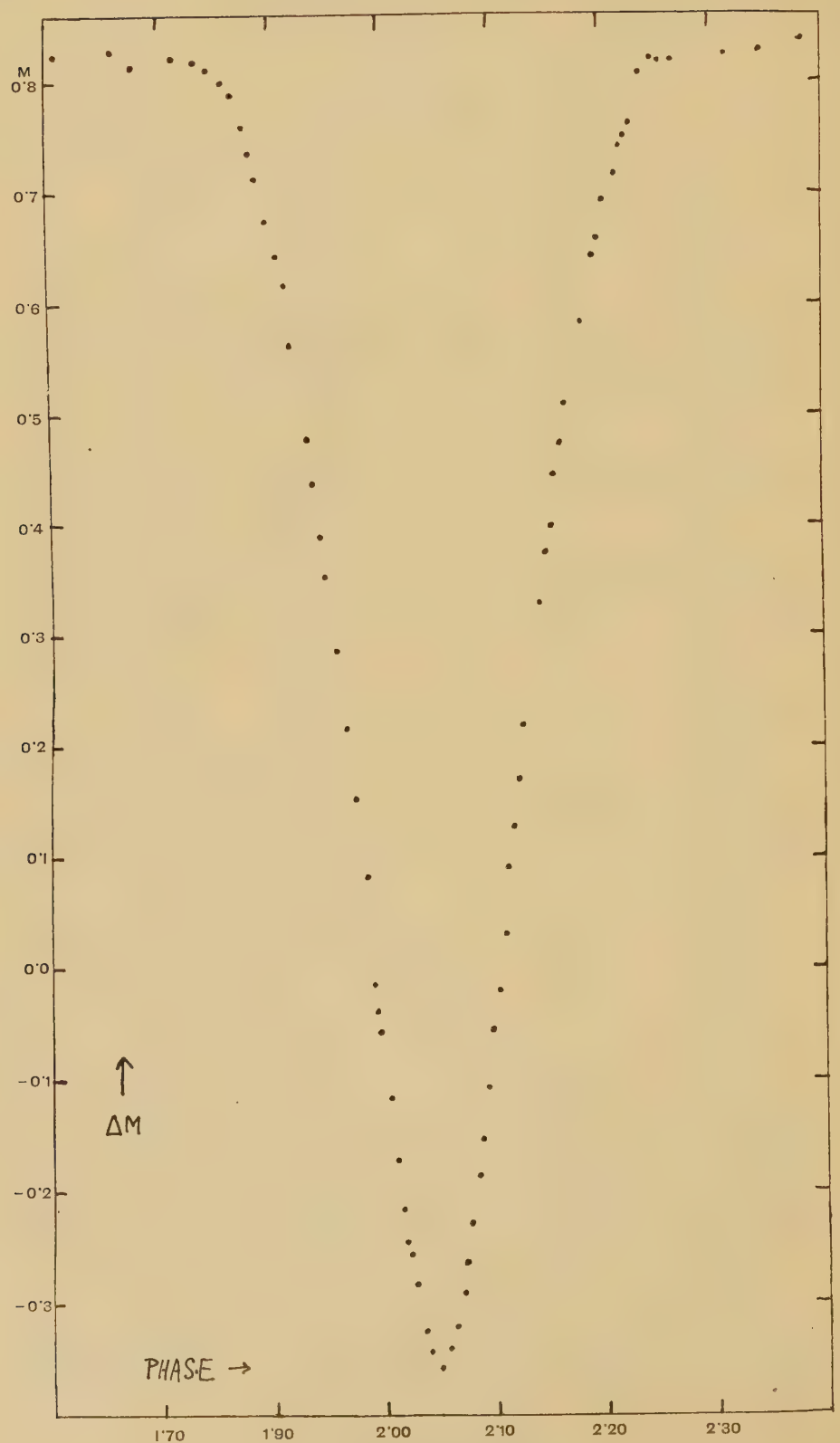


FIG. 2.—*Principal minimum.*

Phase	ΔM	No. of Observations	Phase	ΔM	No. of Observations
1·9111	+0·616	1	2·464	+0·831	4
1·9194	·563	3	2·604	·832	5
1·9316	·477	3	2·655	·841	9
1·9375	·437	3	2·689	·835	2
1·9440	·389	1	2·714	·841	5
1·9484	·353	1	2·765	·837	4
1·9580	·286	2	2·808	·834	6
1·9668	·214	3	2·857	·842	7

considerations referred to in section 3. With McLaughlin's elements, the eclipsing pair are most remote relative to the common centre of mass, G, of the triple system, at 1931·93. Also, 239 periods of the binary are equivalent to one period of Algol around G, so that in the course of one period of the eclipsing system Algol moves about $1\frac{1}{2}$ degrees around G. If the orbital motion of Algol around G is parallel to the line of sight and away from us, the observed period should exceed the mean period by

$$\frac{3}{2} \cdot \frac{\pi}{180} \cdot 5 \text{ minutes},$$

or, approximately,

$$0^{d}\cdot00010.$$

The actual increase of $2^{d}\cdot86757$ over the mean period is a little more than $2\frac{1}{2}$ times this difference. But this does not represent the whole of the

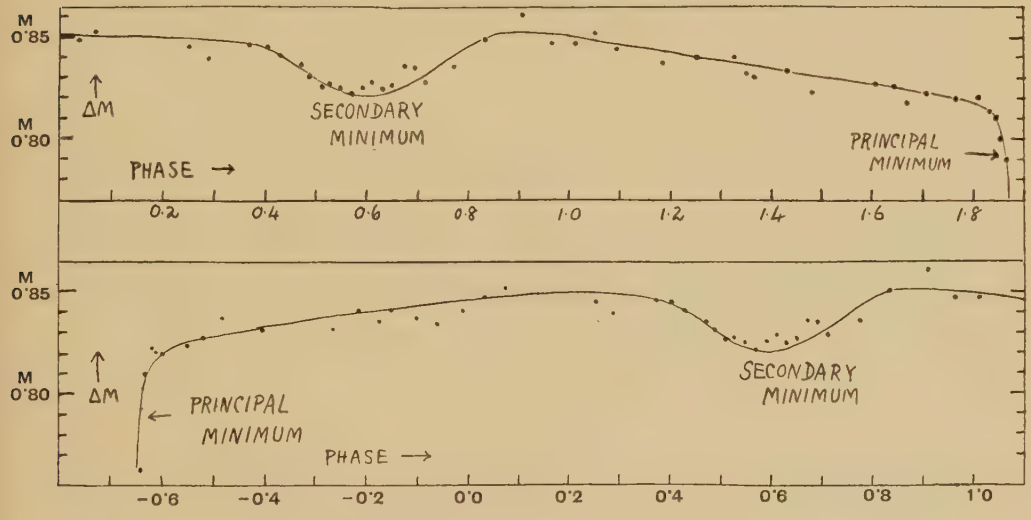


FIG. 3.—*Secondary minimum.*

discrepancy for, according to McLaughlin's elements, the motion of Algol around G at the mean time of the observations on which the period of $2^{d}\cdot86757$ is based is inclined to the line of sight by about 60°; the discordance is thus larger by a factor sec 60° or 2.

The observational material in this paper is unfortunately not sufficient for further determinations of the systematic changes in the period of Algol.

More continuous observations of the principal minimum are desirable in order to study the changes in period in greater detail.

7. The remainder of the paper consists of Table IV, which gives the normal places for the light-curve, and the figures 2 and 3, which show the normal light-curve in detail.

Observatory, Cambridge :
1937 *March* 3.

THE MEAN MOTIONS OF THE LUNAR PERIGEE AND NODE AND THE FIGURE OF THE MOON.

H. Spencer Jones, F.R.S., Astronomer Royal.

In a recent paper * E. W. Brown has found that certain terms neglected in his original computations of the theoretical motions of the lunar perigee and node are not negligible. The terms in the motion of the perigee with factor e'^4 had not been computed, because the first term arranged along powers of m, given by Delaunay, contributed only 1″. But the effect of subsequent terms, which have large coefficients, is not negligible, and the whole series is estimated to contribute $+11'' \pm 2''$ to the centennial motion of the perigee. The terms of the sixth order with respect to e, γ, which were computed neither by Delaunay nor by Brown, add $+3''$ to the motion of the perigee and $-4''$ to the motion of the node.

E. W. Brown and W. J. Eckert have a new numerical investigation in progress which it is hoped will give the solar portions of the centennial motions accurately to 1″. Meanwhile it is of interest to examine what effect these preliminary revisions have on the deduced figure of the Moon, because previous investigators have found difficulties in reconciling the evidence derived in this way as to the figure of the Moon with other evidence.

The motions of the perigee and node contain terms due to solar and planetary actions, the mass of the Earth, the figures of the Earth and Moon and a relativity correction. The figure of the Earth terms depend upon the ellipticity of the Earth. These are computed for an ellipticity of 1/297, but terms in δk are included to correspond to an ellipticity of $1/(297+\delta k)$. We have then the following values :— †

	Perigee		Node	
	″	″	″	″
Principal solar action	+14642692·	± 3	−6967204·	± 3
Planetary action	+ 253·2	± 1	− 137·4	± 1
Mass of the Earth	− 68·5		+ 18·5	
Neglected terms	+ 14·	± 2	− 4·	± 1
Relativity correction	+ 2·0		+ 1·9	
Figure of the Earth	+ 638·3 − 4·42δk		− 597·4 + 4·13δk	
Total	+14643531· − 4·42δk	± 4	−6967922· + 4·13δk	± 3
Observed ‡	+14643535·	± 1	−6967943·	± 2
Difference (figure of the Moon)	+ 4″ + 4″·42δk	± 4″	− 21″ − 4″·13δk	± 3″

* *M.N.*, **97**, 116, 1936. † See, e.g., *Cape Annals*, **13**, pt. iii, 47, 1932.

‡ The observed values are the means of concordant results from meridian observations and occultations (*Cape Annals*, **13**, pt. iii, 45, 1932).

The differences between the observed and theoretical values may be attributed to the effect of the figure of the Moon, as figure of the Moon terms have been omitted from the theoretical values given above.

The ratios of the portions of the motions due to the figure of the Moon, to the portions due to the figure of the Earth are, for the perigee and node respectively,

$$\frac{b^2}{b'^2}\cdot\frac{\mathcal{J}'\rho_e+\frac{3}{2}K'\rho_c}{\mathcal{J}(1-\frac{3}{2}\sin^2\epsilon_1)\rho_e} \quad \text{and} \quad \frac{b^2}{b'^2}\cdot\frac{(\mathcal{J}'+\frac{1}{2}K')\sin 2(i+i_1)}{\mathcal{J}(1-\frac{3}{2}\sin^2\epsilon_1)\sin 2i},$$

where ρ_e, ρ_c are constants having the values $\rho_e=+0{\cdot}386$, $\rho_c=-0{\cdot}678$ and i, i_1 are respectively the inclination of the Moon's orbit and equator to the ecliptic, having the values $i=5^\circ\,8'{\cdot}5$, $i_1=1^\circ\,32'{\cdot}1$.

$\mathcal{J}'$, K' are the quantities used by de Sitter and are defined by

$$\mathcal{J}'=\frac{3}{2}\cdot\frac{2C'-A'-B'}{2Mb'^2} \qquad K'=\frac{3}{2}\cdot\frac{B'-A'}{Mb'^2}.$$

The quantity $\mathcal{J}$ depends upon the ellipticity of the Earth's figure:

$$\mathcal{J}=1/(297+\delta k)-0{\cdot}0017287.$$

b, b' denote the largest radii of the Earth and Moon. A', B', C' are moments of inertia of the Moon about the axis pointing to the Earth, the axis in the direction of motion and the axis of rotation respectively.

Combining these expressions with the motions due to the figure of the Earth as given above, and making the numerical substitutions, we find for the motions due to the figure of the Moon:

$$d\Pi=+38030''\mathcal{J}'-100150''K'=+\ 4+4{\cdot}42\delta k\pm 4. \qquad (1)$$

$$d\Omega=-46050\ \mathcal{J}'-\ 23025\ K'=-21-4{\cdot}13\delta k\pm 4. \qquad (2)$$

It may be noted that the δk portion of $\mathcal{J}$ annuls the δk portions of the motions due to the figure of the Earth.

There is another relationship involving $\mathcal{J}'$, K' given by Hayn, between β and the mean inclination of the Moon's orbit to the ecliptic:

$$\beta(1+0{\cdot}0047f)=0{\cdot}0006286,$$

where

$$\beta=\frac{C'-A'}{C'}; \qquad \alpha=\frac{C'-B'}{C'}; \qquad f=\frac{\alpha}{\beta}.$$

If we put $g'=3C'/2M'b'^2$ we have

$$g'\beta=\mathcal{J}'+\tfrac{1}{2}K'; \qquad g'\alpha=\mathcal{J}'-\tfrac{1}{2}K',$$

and therefore

$$1{\cdot}0047\mathcal{J}'+0{\cdot}4976K'=0{\cdot}0006286g'. \qquad (3)$$

Jeffreys * has recently shown that g' can be computed with very small uncertainty. He obtains the value $g'=0{\cdot}596\pm 0{\cdot}001$, which is nearly equal to the limiting value of 0·60 that corresponds to a homogeneous Moon.

* *M.N., Geophys. Suppl.*, 4, 5, 1937.

We have therefore the relationship

$$1{\cdot}0047\mathcal{J}' + 0{\cdot}4976K' = 0{\cdot}0003746 \pm 0{\cdot}0000011. \tag{4}$$

When (1), (2) are solved we obtain

$$\left.\begin{aligned} \mathcal{J}' &= +{\cdot}000400 + {\cdot}000094\delta k, \\ K' &= +{\cdot}000112 - {\cdot}000008\delta k, \end{aligned}\right\} \tag{5}$$

showing that K' is nearly independent of the Earth's ellipticity.

When (1), (2), (4) are combined we obtain

$$\left.\begin{aligned} \mathcal{J}' &= +0{\cdot}0003136 \pm 0{\cdot}000021, \\ K' &= +0{\cdot}0001198 \pm 0{\cdot}000040, \\ \delta k &= -0{\cdot}92 \pm 0{\cdot}95. \end{aligned}\right\} \tag{6}$$

The correction of the mean motions of the perigee and node for the hitherto neglected terms has decreased the value of $\mathcal{J}'$ and increased that of K'. The value of K' is positive (as it must be for stability) and appreciably larger than its probable error. The value derived from previous discussions has been smaller than its probable error.

The value of $f = (\mathcal{J}' - \frac{1}{2}K')/(\mathcal{J}' + \frac{1}{2}K')$ from (5) is

$$f = 0{\cdot}776 + {\cdot}088\delta k.$$

All available evidence indicates that δk has a zero or a small negative value. Hence the maximum value for f will be 0·78. If we use the values of $\mathcal{J}'$, K' given in (6) we obtain

$$f = 0{\cdot}68 \pm 0{\cdot}09.$$

It is satisfactory to find that the value of f is now in good agreement with the values deduced from the librations in longitude. Franz and Stratton obtained 0·49 and 0·50 respectively from the discussion of Schlüter's observations. Hayn obtained a value of 0·75 and, later, one of 0·85, and attributed the smaller values of other investigators to errors in the adopted radius vector of Mösting A. Völkel obtained 0·78.* The value inferred from previous investigations of the mean motions of the Moon's perigee and node has been about 0·96 or 0·97,† appreciably larger than any of the observed values and not much smaller than the theoretically maximum possible value of 1·00.

The value of the ellipticity obtained above is

$$1/e = 296{\cdot}08 \pm 0{\cdot}95.$$

This value is rather smaller than the value of 296·38 recently derived by Jeffreys, who used the motions (unrevised) of the lunar perigee and node

* *Note added in Proof.*—Völkel's value was based on the discussion of heliometer observations of Mösting A made by Michailowski in Kazan, 1898–1905. A recent more rigid discussion of these observations by Belkowitsch (*Engelhardt Observatory Bulletin*, No. 10, 1936) gives $f = 0{\cdot}84 \pm 0{\cdot}08$.

† *E.g.* H. Spencer Jones, *Cape Annals*, **13**, pt. iii, 51, 1932; H. Jeffreys, *M.N., Geophys. Suppl.*, **4**, 7, 1937.

in combination with other material, though it agrees within the probable errors. From geodetic measures in the United States Hayford obtained 297·0 ± 0·5 ; from observations of the intensity of gravity Helmert obtained 296·7 ± 0·4. These values all agree within their probable errors and are larger than the value of 294·0 adopted by Brown in the construction of his *Tables of the Moon.*

It is worth notice that if the values of $\mathcal{J}'$, K' in (5) are substituted in (3) to deduce g', the value obtained is

$$g' = 0{\cdot}728 + 0{\cdot}143\delta k. \qquad (7)$$

The unrevised values of the motions of the perigee and node, used in my discussion (*loc. cit.*) in 1932, gave a value for the constant term of 0·867. Unless the density of the Moon decreases towards the centre, g' cannot exceed 0·60, so that the value obtained in the present discussion is more probable. The substitution of δk from (6) in (7) gives, of course, the value of g' that was used to obtain (4).

Summary.—Brown has found that terms of the fourth and sixth orders hitherto not taken into account in the theoretical computations of the mean motions of the lunar perigee and node are not negligible. These corrections have been applied and a reinvestigation made of the figure of the Moon. The results now give agreement between the inferred value of the quantity f and the value deduced from libration observations. The ellipticity of the Earth is found to be 1/296·08 ± 0·95.

OCCULTATIONS OBSERVED AT THE ROYAL OBSERVATORY, CAPE OF GOOD HOPE, IN 1936.

(Communicated by H.M. Astronomer)

The occultations were reduced in the same way as in previous years (see *M.N.*, **96**, 808, 1936).

The adopted longitude of the 8-inch transit circle has been changed from $1^h\ 13^m\ 54^s{\cdot}70$ to $1^h\ 13^m\ 54^s{\cdot}60$ E.

A few observations have been made with the 18-inch and 8-inch refractors of the Victoria telescope, for which the adopted position is:

Longitude	$1^h\ 13^m\ 54^s{\cdot}57$
Latitude	$-33^\circ\ 56'\ 6''{\cdot}8$
Log $\rho \cos \phi'$	9·91936
Log $\rho \sin \phi'$	9·74436n.

A correction of $+3''{\cdot}0$ has been added to the Moon's mean longitude as given in the *Nautical Almanac.*

An analysis of the equations of condition—rejecting No. 13—gives:

Mean Epoch	$\Delta\lambda$	Wt.	$\Delta\beta$	Wt.
1936·52	$+2''{\cdot}76$	52·8	$-0''{\cdot}75$	22·9

Using a value of $\Delta\lambda$ varying uniformly from $+3''\cdot20$ at 1936·0 to $+2''\cdot80$ at 1937·0 and $\Delta\beta = -0''\cdot50$, the probable error of a single observation is $\pm 0''\cdot74$.

No.	Date	Star	Mag.	Phase	Instrument	Obs.	U.T. h m s
	1936						
1	Jan. 20	19 Scorpii	4·8	R.D.	Heliom.	A.W.	2 48 25·47
2	,, 30	B.D. +18° 283	7·1	D.D.	18-inch	J.	18 23 46·71
3	,, 30	,,	,,	,,	10 ,,	S.	46·81
4	,, 30	,,	,,	,,	Heliom.	A.W.	46·81
5	,, 31	B.D. +22° 457	6·9	D.D.	,,	A.W.	19 33 50·93
6	Feb. 14	43 H Virginis	5·6	R.D.	,,	S.	2 22 49·38
7	,, 17	Co. D. −24° 12857	7·0	R.D.	,,	S.	0 11 10·05
8	,, 18	Co. D. −24° 13521	7·1	R.D.	,,	S.	1 31 25·52
9	,, 28	B.D. +24° 599	6·4	D.D.	10-inch	H.P.	19 19 13·58
10	Mar. 2	87 B Geminorum	5·8	D.D.	,,	J.	18 36 10·03
11	,, 2	,,	,,	,,	Heliom.	S.	10·03
12	,, 2	,,	,,	,,	10-inch	H.P.	10·03
13	,, 15	44 Ophiuchi	4·3	D.B.	Heliom.	A.W.	22 59 22·57
14	,, 15	191 B Ophiuchi	6·3	R.D.	,,	A.W.	23 13 5·02
15	,, 15	44 Ophiuchi	4·3	R.D.	,,	A.W.	23 36 19·40
16	,, 29	B.D. +23° 1433	6·8	D.D.	10-inch	J.	19 56 48·13
17	,, 29	,,	,,	,,	Heliom.	A.W.	47·93
18	Apr. 13	1 Sagittarii	5·1	R.D.	,,	A.W.	3 7 20·75
19	,, 14	*o* Sagittarii	3·9	D.B.	,,	A.W.	0 1 31·25
20	,, 14	B.D. −21° 5233	6·9	R.D.	,,	A.W.	0 53 3·27
21	,, 14	*o* Sagittarii	3·9	R.D.	,,	A.W.	1 15 54·12
22	,, 17	B.D. −9° 5854	6·8	R.D.	,,	A.W.	2 34 16·79
23	,, 25	5 Geminorum	5·9	D.D.	10-inch	S.	16 43 52·47
24	,, 25	,,	,,	,,	Heliom.	A.W.	52·67
25	,, 27	10 H Cancri	6·1	D.D.	10-inch	J.	17 25 24·06
26	,, 27	,,	,,	,,	Heliom.	A.W.	23·96
27	May 1	p^5 Leonis	5·4	D.D.	,,	A.W.	19 43 17·54
28	,, 10	4 Sagittarii	4·8	R.D.	,,	A.W.	4 28 6·56
29	,, 15	51 Aquarii	5·8	R.D.	,,	A.W.	1 40 32·92
30	,, 23	B.D. +22° 1456	6·8	D.D.	,,	A.W.	16 55 16·45
31	June 9	31 B Capricorni	6·4	R.D.	,,	A.W.	3 47 26·01
32	,, 21	B.D. +18° 1880	8·4	D.D.	10-inch	S.	16 38 20·38
33	,, 21	B.D. +18° 1882	6·4	D.D.	,,	S.	16 46 38·11
34	,, 21	,,	,,	,,	Heliom.	A.W.	38·41
35	,, 28	85 Virginis	6·2	D.D.	10-inch	S.	21 14 47·96
36	,, 28	,,	,,	,,	Heliom.	A.W.	46·96
37	July 2	ρ Ophiuchi	4·8	D.D.	,,	A.W.	1 30 13·91
38	,, 2	39 Ophiuchi *comes*	6·9	D.D.	,,	A.W.	23 21 9·20
39	,, 2	39 Ophiuchi	5·4	D.D.	,,	A.W.	23 21 27·25
40	,, 4	π Sagittarii	3·0	D.B.	,,	A.W.	22 2 55·83
41	,, 4	π Sagittarii	3·0	R.B.	,,	A.W.	23 10 39·29
42	,, 13	26 Arietis	6·1	R.D.	,,	A.W.	4 31 47·72
43	,, 14	66 Arietis	6·1	R.D.	,,	A.W.	3 27 13·64
44	Aug. 1	33 Sagittarii	5·8	D.D.	,,	A.W.	1 51 48·16
45	,, 7	*d* Piscium	5·6	R.D.	,,	A.W.	1 43 55·86
46	,, 24	B.D. −21° 4152	7·0	D.D.	10-inch	S.	19 58 13·04
47	,, 24	,,	,,	,,	Heliom.	A.W.	13·24
48	,, 25	ρ Ophiuchi, *S.f.*	5·2	D.D.	10-inch	S.	16 51 10·25
49	,, 25	,, *N.pr.*	5·7	,,	,,	S.	14·44
50	,, 25	,, ,,	,,	,,	Heliom.	A.W.	15·23

No.	Date	Star	Mag.	Phase	Instrument	Obs.	U.T.
	1936						h m s
51	Aug. 25	Co. D. −23° 12862	8·0	D.D.	Heliom.	A.W.	16 59 12·73
52	Sept. 8	315 B Tauri	6·3	R.D.	,,	A.W.	1 23 1·14
53	,, 23	21 G Sagittarii	5·7	D.D.	10-inch	S.	21 4 14·46
54	,, 23	Co. D. −23° 13804	7·0	D.D.	,,	S.	21 10 37·71
55	,, 23	Co. D. −22° 12457	7·8	D.D.	,,	S.	21 13 25·55
56	,, 23	B.D. −22° 4533	6·9	D.D.	,,	S.	21 54 43·96
57	,, 23	,,	,,	,,	Heliom.	A.W.	44·26
58	,, 24	B.D. −21° 5200	8·6	D.D.	,,	A.W.	21 33 14·49
59	,, 24	ξ Sagittarii	3·6	D.D.	10-inch	S.	21 36 59·87
60	,, 24	,,	,,	,,	Heliom.	A.W.	37 0·47
61	Oct. 2	η Piscium	3·7	D.B.	,,	A.W.	1 53 51·56
62	,, 2	η Piscium	3·7	R.D.	,,	A.W.	3 1 5·51
63	,, 3	26 Arietis	6·1	R.D.	,,	A.W.	1 43 17·95
64	,, 8	B.D. +20° 1856	6·8	R.D.	,,	A.W.	2 16 42·69
65	,, 21	115 B Sagittarii	5·8	D.D.	,,	A.W.	20 13 22·92
66	Nov. 3	9 Geminorum	6·3	R.D.	,,	A.W.	0 55 50·81
67	,, 5	ζ′ Cancri	5·1	R.D.	,,	A.W.	1 15 37·07
68	,, 5	B.D. +18° 1882	6·4	R.D.	,,	A.W.	2 26 11·18
69	Dec. 5	155 B Leonis	6·5	R.D.	,,	A.W.	1 5 25·65
70	,, 24	ϵ Arietis, *N.f.*	4·6	D.D.	18-inch	J.	19 25 49·55
71	,, 24	,,	,,	,,	Heliom.	A.W.	49·55
72	,, 25	B.D. +22° 617	6·9	D.D.	,,	A.W.	18 30 10·66
73	,, 26	103 Tauri	5·5	D.D.	8-inch	J.	19 31 36·76
74	,, 26	,,	5·5	,,	Heliom.	A.W.	36·56
75	,, 29	ζ′ Cancri	5·1	R.D.	,,	A.W.	20 35 29·26
76	,, 31	89 B Leonis	6·3	R.D.	,,	A.W.	21 20 20·99
77	,, 31	π Leonis	4·9	R.D.	,,	A.W.	22 40 30·93

Notes

6. Observation of doubtful value.
7. ,, ,, ,, ; thick cloud passing.
13. Very difficult.
35. Observer S., not good observation.
40. Difficult.
65. Through cloud.
72. Cloudy ; very faint and difficult.

No.	Star's Apparent R.A.	Star's Apparent Dec.	Equations of Condition
	h m s	° ′ ″	″
1	16 16 47·40	−24 1 1·9	$+0·51 = -·865\Delta\alpha + ·312\Delta\delta = - ·977\Delta\lambda + ·212\Delta\beta$
2	2 10 19·70	+19 19 5·9	$-0·80 = +·327 + ·939 = + ·610 + ·793$
3	...	...	... ...
4	...	...	... ...
5	3 10 35·06	+22 43 7·2	$+2·20 = +·868 - ·351 = + ·841 - ·541$
6	14 11 54·16	−17 54 21·2	$-2·48 = -·892 + ·345 = - ·999 + ·049$
7	16 45 51·82	−24 31 51·9	$+0·85 = -·718 - ·612 = - ·758 - ·652$
8	17 46 3·58	−24 11 18·4	$+0·56 = -·676 + ·668 = - ·704 + ·710$
9	3 53 38·09	+24 16 48·3	$+0·90 = +·845 - ·385 = + ·860 - ·510$
10	6 48 9·47	+23 40 44·0	$-0·49 = +·781 - ·530 = + ·926 - ·379$
11	...	...	... ...
12	...	...	... ...

No.	Star's Apparent R.A. h m s	Dec. ° ′ ″	Equations of Condition ″
13	17 22 29·65	−24 7 8·7	(+5·46) = +·645Δα + ·704Δδ = + ·718Δλ + ·696Δβ
14	17 21 13·47	−24 11 16·1	+0·17 = −·784 + ·505 = − ·857 + ·516
15	17 22 29·65	−24 7 8·7	−2·24 = −·426 + ·883 = − ·457 + ·890
16	6 34 0·82	+23 39 36·5	+0·47 = +·478 − ·854 = + ·642 − ·767
17	...	...	... ...
18	18 7 51·72	−23 42 58·7	+0·98 = −·884 − ·258 = − ·988 − ·158
19	19 0 53·36	−21 50 11·1	+2·87 = +·925 + ·032 = + ·988 − ·153
20	19 0 33·04	−21 37 35·2	−1·39 = −·653 + ·709 = − ·560 + ·829
21	19 0 53·36	−21 50 11·1	−0·58 = −·914 − ·165 = −1·000 + ·024
22	21 47 12·32	− 9 16 48·5	+0·58 = −·470 − ·879 = − ·779 − ·627
23	6 7 38·22	+24 26 11·3	−0·37 = +·878 − ·281 = + ·983 − ·181
24	...	...	... ...
25	8 1 5·43	+19 1 22·6	+0·64 = +·670 − ·708 = + ·876 − ·481
26	...	...	... ...
27	11 10 31·65	+ 0 16 26·1	+0·49 = +·317 − ·948 = + ·681 − ·733
28	17 55 56·62	−23 48 43·7	−0·40 = −·491 − ·844 = − ·609 − ·793
29	22 20 48·84	− 5 9 32·0	−0·97 = −·994 − ·044 = − ·932 + ·361
30	6 41 5·64	+22 54 13·3	−1·14 = +·002 −1·000 = + ·167 − ·986
31	20 25 10·87	−15 57 5·4	−0·19 = −·960 − ·031 = − ·959 + ·285
32	8 10 23·44	+17 55 13·2	+1·42 = +·754 − ·613 = + ·936 − ·351
33	8 10 33·02	+17 52 7·4	+0·99 = +·685 − ·696 = + ·892 − ·453
34	...	...	... ...
35	13 42 11·21	−15 27 8·0	−0·08 = +·962 − ·023 = + ·954 + ·299
36	...	...	... ...
37	16 21 48·86	−23 18 13·3	−1·75 = +·415 + ·891 = + ·382 + ·924
38	17 14 10·75	−24 13 2·6	−2·51 = +·822 − ·431 = + ·897 − ·442
39	17 14 10·83	−24 13 13·3	−2·36 = +·817 − ·444 = + ·891 − ·454
40	19 6 2·04	−21 7 30·2	+1·99 = +·504 + ·840 = + ·699 + ·715
41	19 6 2·04	−21 7 30·2	−0·58 = −·857 + ·390 = − ·822 + ·570
42	2 27 5·29	+19 34 29·6	+1·16 = −·741 − ·622 = − ·921 − ·390
43	3 24 44·10	+22 35 12·7	+0·87 = −·771 − ·555 = − ·919 − ·393
44	18 50 15·34	−21 26 17·7	−0·36 = +·504 + ·839 = + ·686 + ·728
45	0 17 21·89	+ 7 50 29·0	−0·79 = +·017 −1·000 = − ·377 − ·926
46	15 33 49·96	−21 54 37·9	−1·11 = +·893 + ·262 = + ·909 + ·416
47	...	...	... ...
48	16 21 48·33	−23 18 12·7	−0·96 = +·760 + ·559 = + ·778 + ·628
49	...	...	... ...
50	...	...	... ...
51	16 21 48·48	−23 15 42·2	+0·58 = +·611 + ·745 = + ·600 + ·800
52	4 52 25·30	+24 29 35·9	+0·11 = −·757 − ·560 = − ·847 − ·532
53	17 58 5·49	−22 46 51·0	−2·32 = +·643 + ·714 = + ·760 + ·650
54	17 58 34·27	−23 1 53·9	−2·05 = +·870 − ·321 = + ·915 − ·404
55	17 58 42·67	−22 54 23·4	+0·42 = +·908 + ·149 = + ·998 + ·060
56	18 0 6·95	−22 50 26·2	+0·15 = +·916 + ·080 = +1·000 − ·011
57	...	...	... ...
58	18 53 51·91	−21 14 6·9	−0·85 = +·929 − ·044 = + ·975 − ·224
59	18 53 58·85	−21 11 27·0	+0·81 = +·926 + ·088 = + ·996 − ·093
60	...	...	... ...
61	1 28 8·01	+15 1 26·3	−0·23 = +·741 + ·645 = + ·934 + ·356
62	1 28 8·01	+15 1 26·3	+0·10 = −·968 − ·041 = − ·957 + ·289
63	2 27 7·59	+19 34 41·5	−0·36 = −·663 + ·715 = − ·490 + ·872
64	7 33 23·45	+20 18 1·9	+1·44 = −·924 − ·195 = − ·908 − ·419
65	18 34 7·94	−21 27 6·4	−2·53 = +·519 + ·829 = + ·678 + ·735

No.	Star's Apparent R.A.	Star's Apparent Dec.	Equations of Condition
	h m s	° ′ ″	″
66	6 13 9·53	+23 45 49·4	$-0\cdot41 = -\cdot460\Delta\alpha + \cdot867\Delta\delta = -\cdot592\Delta\lambda + \cdot806\Delta\beta$
67	8 8 36·87	+17 50 18·6	$+2\cdot22 = -\cdot954 - \cdot054 = -\cdot943 - \cdot334$
68	8 10 35·70	+17 51 59·6	$+0\cdot56 = -\cdot757 + \cdot612 = -\cdot933 + \cdot361$
69	10 19 59·36	+ 6 0 47·4	$+0\cdot58 = -\cdot499 + \cdot866 = -\cdot798 + \cdot602$
70	2 55 38·37	+21 5 31·8	$-1\cdot02 = +\cdot462 + \cdot871 = +\cdot668 + \cdot744$
71	...	...	... ...
72	3 57 14·44	+23 1 37·7	$+2\cdot07 = +\cdot527 - \cdot822 = +\cdot464 - \cdot886$
73	5 4 18·74	+24 11 3·5	$-0\cdot09 = +\cdot851 - \cdot375 = +\cdot924 - \cdot383$
74	...	...	... ...
75	8 8 38·48	+17 50 12·2	$+1\cdot48 = -\cdot883 + \cdot384 = -\cdot994 + \cdot108$
76	9 54 49·44	+ 8 36 45·3	$+1\cdot04 = -\cdot743 + \cdot661 = -\cdot944 + \cdot329$
77	9 56 55·12	+ 8 20 39·6	$+0\cdot57 = -\cdot820 + \cdot562 = -\cdot978 + \cdot207$

Star Places

32, 38, 51, 55, 58. From means of various catalogues.
53, 65. From *A.E.*, 1936.
70, 71. $+0^{s}\cdot02$, $+0''\cdot68$ applied to position of mass.

OBSERVATIONS OF OCCULTATIONS MADE AT TEDDINGTON DURING 1936.

John L. Haughton, D.Sc.

The observations were made as previously described, a correction for lag being applied in Col. 6. The amounts subtracted are 0·2 sec. for Nos. 88, 89, 91, 94 and 101 ; 0·5 sec. for Nos. 85, 86 and 93 ; and 1·0 sec. for Nos. 92, 98 and 99. No. 97 was so uncertain that no estimate of the lag was made.

No.	Date	Star	Mag.	Phase	U.T.	Prob. Error	Remarks
	1936				h m s		
85	Feb. 3	9 Gem	6·3	D	22 19 08·2	± 0·4	Very difficult.
86	4	36 B Gem	6·0	D	02 45 01·0	0·4	,,
88	27	47 Are	5·8	D	18 51 59·7	0·3	
89	28	33 Tau	6·0	D	18 26 44·3	0·3	
91	Apr. 28	B.D. +13° 2021	6·8	D	19 56 44·5	0·3	
92	Aug. 10	B.D. +20° 541	6·7	R	01 11 55·2	0·6	Difficult.
93	Sept. 27	137 B Cap	6·2	D	20 18 37·5	0·3	
94	28	κ Aqu	5·3	D	23 02 41·3	0·3	
97	Oct. 31	A Tau	4·5	D	21 56 21·0	?	Very difficult.
98	31	A Tau	4·5	R	22 55 19·5	?	,,
99	31	B.D. +21° 587 (?)	6·0	R	23 01 15·4	?	,,
101	Nov. 21	$c2$ Cap	6·2	D	16 40 07·6	0·3	

Notes.—No. 97. A disappearance on the bright limb ; unreliable.
Nos. 98 and 99. Both doubtful. No. 99 not given in *N.A.*